Accommodations for Learning

From Making Tens to Place Value

For students to become effective problem solvers, they need to be able to reason numerically. Knowing the combinations of 10 provides a good foundation for decomposing and composing numbers flexibly and solving problems efficiently. This is a goal that Stephanie Clements, a math resource teacher, has for all her students.

It is March and despite the fact that Emily has had plenty of opportunities to play games to learn the combinations of 10, she continues to rely on her fingers to solve these combinations. Ms. Clements wonders what might help Emily overcome this difficulty. She decides to work with Emily individually to better understand how she is thinking about number combinations.

I have decided to work with Emily, who relies mostly on her fingers to solve combinations of one-digit numbers that sum to 10. At this time in the school year, I expect second graders to know these combinations by heart. But Emily has not yet begun to develop efficient strategies for figuring out these problems. I have always wondered what it is that students need to experience to really know the number combinations that make a total of 10 without relying on their fingers to count.

Emily has played many games that foster practice with combinations of 10—*Tens Go Fish, Dot Addition,* and *Quick Images*—some of which come from the first-grade *Investigations* units. Yet the practice has not been enough. This time I intended to move beyond the practice and learn more about how Emily thinks.

I gave her a word problem that involved adding 6 and 4. When Emily explained how she solved it, she said: "I started from 6 and counted on 4 to make 10." It was clear that she knew how to count on from the correct number and when to stop counting. Then we made a list of equations using one-digit numbers that total 10. We had all the possible combinations. I asked her if she noticed anything about the number combinations on the list. She saw that $2 + 8$ was the same as $8 + 2$. I took this opportunity to show how equal signs communicate that idea. So she made a new list of all the combinations of 10 like this:

$$8 + 2 = 2 + 8 = 10$$

$$6 + 4 = 4 + 6 = 10$$

$$7 + 3 = 3 + 7 = 10$$

$$10 + 0 = 0 + 10 = 10$$

$$9 + 1 = 1 + 9 = 10$$

$$5 + 5 = 5 + 5 = 10$$

After this review, with the combinations of 10 fresh in her mind, I played a new game with her, using the number cards. I put a card down, and Emily had to figure out what number was needed to make 10. She continued to count on, using her fingers. As the game continued, I noticed that she was getting better at giving the numbers needed to make 10. I asked her how she was thinking about it. She explained to me that when she was shown a card with a 3, instead of counting on, she subtracted it from her 10 fingers, thus coming up with 7 easily. She continued to explain that when the number given was 1, 2, 3, 4, or 5, it was easier for her to subtract than to count on.

I thought it was interesting that she discovered she could use either addition (counting on) or subtraction to figure out her answers. She was definitely working on an important idea about the relationship between the whole and its parts. She knew that if one of the parts was 3, and the whole was 10, she could subtract 3 to find the other part, 7. It was also interesting to note that she was not using counting back when she was subtracting. She simply bent her fingers to show the numeral on the card and immediately saw that there were 7 fingers left. She does not need to count the remaining fingers; she can see them. She is definitely developing important ideas about number relationships.

At the end of the session, Emily felt good about herself and seemed confident that she had figured out a way to find combinations of numbers that make 10. I plan to build on what she knows about 10 to work on addition with sums greater than 10, while I continue to help her strengthen her knowledge of combinations that make 10.

In this case, Ms. Clements works with Emily to understand her thinking so that she can help her become more fluent with the combinations of 10. Through their work together, Ms. Clements discovers some of Emily's strengths and is able to build on them. At the end of the session, Emily feels more confident in herself as a learner.

Questions for Discussion

1. What experiences did Ms. Clements provide for Emily to build her numerical reasoning skills and what does she observe about Emily's understanding?
2. What were the mathematical ideas that Emily was working on during this session?
3. What strategies have you used to help students become more fluent who, like Emily, are struggling with combinations of 10?

Tens and More

In this case, Stephanie Clements, a second-grade math resource teacher, continues her work with Emily to help her become a more efficient problem solver. In their last session together, Ms. Clements worked with Emily on combinations of two numbers that sum to 10. In this session, Ms. Clements wants to see if Emily can use the knowledge she is developing to solve more complex problems.

In this meeting with Emily, I wanted to build on her knowledge of number combinations to 10 to solve problems with a total greater than 10. For review, I showed her number cards, and she was to say the number needed to make a 10. She knew her number combinations! She was no longer using her fingers. We then moved on to the focus of my session. I showed her the following problem and asked her how she could use what she knows about tens to solve it.

$9 + 4 =$

She looked at it for a while and then started to count on using her fingers. After she came up with the answer, I asked her if she could solve the problem in another way without using her fingers.

Teacher: What do you need to make the 9 a 10?

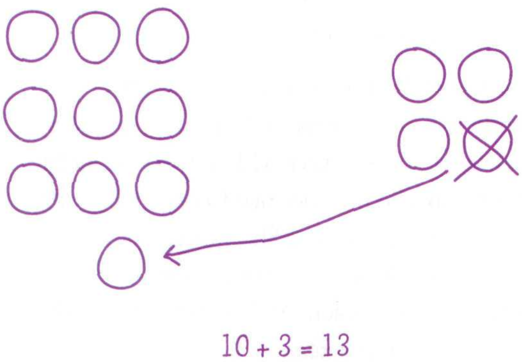
Emily: $9 + 1 = 10$.

Teacher: Where can you get 1 so that the 9 becomes a 10?

Emily: Maybe I can use some from the 4. Then it becomes $10 + 3 = 13$. That's right, that's what I got when I counted on my fingers.

Teacher: Can you draw a picture of what you just did?

Emily drew 9 circles and then 4 circles on her paper. Then she crossed out one of the circles from the group of 4 circles and added a circle to the group of 9 circles, making a group of 10 circles and a group of 3 circles. Underneath, she wrote 10 + 3 = 13.



Teacher: So from your picture, are you saying that 9 + 4 is equal to 10 + 3 and both are equal to 13?

Emily: Yeah, because you see 9 + 4 = 13 and 10 + 3 = 13. What I did was I took part of the 4 to make the 9 a 10, but I still have 3 left, so I did 10 + 3 = 13.

At this point I gave Emily another problem to solve.

8 + 7 =

She wrote 8 + 2 = 10 and then 10 + 5 = 15.

She used the same method of taking apart the smaller number and making the larger number a 10. I gave her more problems of this type, and she solved them the same way.

I reminded her of the meaning of the equal sign, so I wrote 8 + 7 = 10 + 5 = 15. I asked her if this statement made sense. I asked her what the statement means to her. She said, "They all add up to the same amount, and they are equal."

I gave Emily a sheet of paper with problems, and she solved them with ease. I wonder if she will be able to use this understanding when she is presented with two-digit numbers to add, such as 28 + 13. It has been interesting to watch her development from counting all to now making tens to solve problems more efficiently. Above all, she understands it.

Ms. Clements began this session with the clear goal of helping Emily use knowledge she had of combinations to make 10 to solve more complex problems. At first Emily resorted to her comfortable counting on strategy. Ms. Clements responded to this by specifically asking Emily to consider how she could use her knowledge of tens to help her. Ms. Clements helped Emily make a connection to things she knew to solve a new type of problem. As in a previous session, she didn't push Emily to mechanically solve a problem. Instead, she encouraged Emily to use what she already knew so that she could solve the more difficult problems with understanding.

Questions for Discussion

1. What questions did Ms. Clements ask Emily to build her understanding from their previous work together?
2. After this session, how do you think Emily might solve 28 + 13?
3. How do you help your students use knowledge they have to solve more complex problems? What do you do when students have difficulties making those connections by themselves?

Moving on to Bigger Numbers

Stephanie Clements, a second-grade math resource teacher, has been working to help Emily become a more efficient problem solver. In their first two sessions, Emily became more comfortable with the number combinations that make 10 and learned how to use that information to solve problems more easily. She now understands that numbers can be decomposed in different ways and still equal the same amount. For example, she knows that $9 + 6 = 10 + 5 = 15$. *She has started to use this knowledge to create equivalent problems that are easy for her to solve. In this session, Ms. Clements wants to see if Emily can use her new information about numbers to solve two-digit addition problems.*

I started out by giving Emily a couple of problems that I thought she would be able to figure out using her strategy. The first problem I gave her was 16 + 5. I put a box around the problem so that she could clearly differentiate between the problem and her solution to the problem. She immediately wrote 20 + 1 = 21 underneath the boxed expression. I asked her where the numbers 20 and 1 came from.

Emily: I know 6 + 4 = 10, so I need 4 more to make the 16 a 20. So I got the 4 from the 5, and there is still one left. So now it is 20 + 1 = 21.

I thought it was interesting that she did not write an intermediate step like 16 + 4 = 20 to reach 20 + 1 = 21. In her mind she knew that 16 + 4 is 20 and did not feel the need to write it. She went straight to 20 + 1 = 21.

I gave her another problem, 18 + 7. She solved the problem in a similar way as the first by changing the numbers to 20 + 5 = 25. She solved a few more problems similar to these, using a two-digit number plus a one-digit number. Each time, I asked her how she was thinking about the numbers to add. Her strategy was the same, making one of the numbers into a multiple of 10 and adding what was left from the other number.

I decided to give her a problem in which both of the addends were two-digit numbers. The first problem I gave her was 28 + 32. Emily looked at both numbers briefly and quickly wrote 30 + 30 underneath the box where I had put the problem.

Emily: That was easy.

Teacher: Why was this easy?

Emily: All I needed to do was to add 2 to 28 to get it to 30. I had 30 left after I took the 2 away from 32 to give to 28, so it is 30 + 30 = 60.

I was excited to see that Emily was able to apply the strategy that she used with the previous numbers to an addition problem involving two-digit numbers for both addends. I gave her several more problems, and she solved them successfully. Then I gave her a problem for which the sum was greater than 100: 77 + 25. As she solved this problem, I asked her to think out loud so that I could understand her steps.

Emily: This is 77, so I need 3 more to get to 80. 25 − 3 = 22; so 80 + 22 = 80 + 20 (from 22) + 2 = 102.

Although she was thinking all these steps, she did not record them all when she wrote her solution to the problem. She just wrote 80 + 22 = 102. I was glad I had asked her to think out loud for me so that I could follow her reasoning.

Next I gave her 69 + 48 to solve. First she changed the problem to 70 + 47 = ___. Then she counted on by 10s from 70 until she had 100. When she got to 100, she thought for a while to figure out what number is 10 more than 100. She looked up at the 100 chart on the wall, but this did not help her. I asked her to count by 1s from 100. When she got to 110, her eyes lit up, and she said, "Oh, 100 and 10 more is 110. Now I get it." Then she wrote 110 + 7 = 117. She did not have any trouble writing 110 or 117.

As I reflect on the strategy that Emily has found success with in solving addition problems, I realize what a powerful strategy she has come to understand. In decomposing and rearranging parts of addends, she is applying her understanding of the basic properties of addition to help her do mental math easily. She has applied her knowledge of numbers that make 10 to bigger numbers. When she said, "I get it" while counting by 1s beyond 100, I wonder what kind of understanding came to her about our number system. I would like to explore this idea with her some more. I feel that the most important learning that Emily has gained is her confidence in solving problems.

Ms. Clements has concentrated her efforts this year on helping Emily build a strong foundation based on understanding number relationships. As a result, Emily has grown from a student who needed to use her fingers to add two single-digit numbers to a student who is able to decompose and compose numbers to efficiently solve two-digit addition problems. In this case, we see the culmination of Ms. Clements's efforts as Emily uses the information she learned in their previous sessions to make the leap to solving more complex math problems. Emily's confidence has grown as well, and she will enter third grade with tools that will help her to continue to build on her numerical reasoning.

Questions for Discussion

1. **In the previous case about Emily, we see her create equivalent problems when adding one-digit numbers with sums greater than 10. What are the mathematical ideas that Emily is working on in this case? How does Ms. Clements help Emily build on her earlier work to solve two-digit addition problems?**

2. **After this session, how do you think Emily might solve 87 + 44?**

3. **Like many students, Emily demonstrates some confusion when counting or adding numbers greater than 100. What are some next steps you might take with Emily or students like her to help them apply their addition strategies to larger numbers?**